# UNIT & DIMENSIONS

## *Preface*

**I**n science, a type of question often asked is How much ? How big ? In order to answer such questions it is important to have systems of measurement which are consistent and understood by all. A unit is a basic division of a measured quantity and it enables to say how much of the quantity we have 10 miles, 2 hours etc. A dimension is a property that can be measured such as distance, time, temperature, speed.

This book consists of theoretical & practical explanations of all the concepts involved in the chapter. Each article followed by a ladder of illustration. At the end of the theory part, there are miscellaneous solved examples which involve the application of multiple concepts of this chapter.

Students are advised to go through all these solved examples in order to develop better understanding of the chapter and to have better grasping level in the class.

Total number of Questions in **Unit & Dimensions** are :

In Chapter Examples ............................................................ 18

Solved Examples .................................................................. 06

**Total no. of questions .......................................................... 24**

## 1. PHYSICAL QUANTITIES

The quantities by means of which we describe the laws of physics are called physical quantities. A physical quantity is completely specified if it has

**(a)** Magnitude only
Ratio
Refractive index, dielectric constant

**(b)** Magnitude and unit
Scalar
Mass, charge, current

**(c)** Magnitude, unit and direction
Vector
Displacement, torque.

**Physical quantity = Magnitude × unit**

**1.1 Quantities :** These are of two types –

**(a)** Fundamental quantities

**(b)** Derived quantities

**(a) Fundamental quantities :**

The quantities which do not depend upon other physical quantities are called fundamental quantities and all other quantities may be expressed in terms of the fundamental quantity.
There are of seven fundamental quantities in SI system-

**(i)** Mass
**(ii)** Length
**(iii)** Time
**(iv)** Temperature
**(v)** Electric current
**(vi)** Luminous intensity
**(vii)** Amount of substance

These quantities are also called base quantities.

**(b) Derived quantities :**
The quantities which are derived with the help of fundamental quantities is called derived quantities as

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}} = \frac{\text{Length}}{\text{Time}}$$

Here we know that length and time are the fundamental quantities.

### Example based on Quantities

**Ex.1** The Bernoulli's equation is given by $P + \frac{1}{2}dv^2 + dgh$ = constant. The quantity $dv^2/2$ has the same units as that of –
(A) Force (B) Impulse
(C) Strain (D) Pressure

**Sol.(D)** Pressure (only quantities with identical dimensions can be added or subtracted)

**Ex.2** Which of the following sets can not enter into the list of fundamental quantities in any system of units ?
(A) Length, mass and velocity
(B) Length, time and velocity
(C) Mass, time and velocity
(D) Length, time and mass

**Sol.(B)** The group of fundamental quantities are those quantities which do not depend upon other physical quantities in the group. But in set (B) we can predict the relation between given quantities as length = velocity × time
Hence set (B) can not enter in to the list of fundamental quantities.
Hence correct answer is (B).

## 2. UNITS

**2.1** That fixed and definite quantity which we take as our **standard of reference** and by which we measure other quantities of same kind, is defined unit. There are of two types.

**(a)** Fundamental Units
**(b)** Derived Units

**(a) Fundamental Units :** The units which are independent and which are not be derived from other units, are defined as fundamental units. e.g. The unit of mass, length, and time.

There are seven fundamental units.

**(i)** Unit of mass
**(ii)** Unit of length
**(iii)** Unit of time
**(iv)** Unit of temperature
**(v)** Unit of electric current
**(vi)** Unit of luminous intensity
**(vii)** Unit of amount of substance

**(b) Standard Units :** The fixed and definite real value of any physical quantity is defined as standard unit.

**2.2 Properties of Units :**

The unit of a physical quantity is inversely proportional to its numerical value i.e., $u \propto \frac{1}{n}$ where u and n are the units of physical quantity and its numerical value respectively. Relation between unit and its numerical value $n_1 u_1 = n_2 u_2$

**2.3 Selection Criteria Of Units :**

**(i)** Selected unit must be universal, of proper size and magnitude

**(ii)** Unit must be not affected by temperature, pressure and time.

**(iii)** Easily definable and reproducible.

**2.4 System Of Units Used :**

These are of Four types –

**(i)** C.G.S – (Centimeter – Gram – Second) system.

**(ii)** M.K.S. – (Metre – Kilogram – Second) system

**(iii)** F.P.S. – (Foot – Pound – Second) system

**(iv)** S.I. – (System – international) system

Following table will show the difference between all the systems.

| Quantity | C.G.S. | M.K.S. | F.P.S. | S.I.(Symb.) |
|---|---|---|---|---|
| Mass | gm | kg | pound | kg |
| Length | cm | m | foot | metre |
| Time | second | sec | sec | second |
| Temperature | kelvin | kelvin | | kelvin (K) |
| Electric Current | | ampere | | ampere(A) |
| Luminous Intensity | | | | candela (cd) |
| Amount of substance | | | | mole(mol) |

**2.5 Some definitions of fundamental units in S.I. system :**

**(a) Metre :** One metre is that distance which accommodates 1, 650, 763.73 waves in vacuum of orange red light emitted by $Kr^{86}$.

**(b) Kilogram :**

**(i)** Kilogram is the mass of platinum iridium cylinder of diameter equal to its height which is preserved in a vault at international Bureau of weights and measures at Severs near Paris.

**(ii)** 1 kilogram is the mass of 1 lit. of water at 4ºC

**(iii)** 1 kilogram is the mass of $5 \times 10^{25}$ atoms of $C^{12}$

**(c) Second :** One second is that amount of time in which a cesium-33 atom makes 9, 192, 631, 770 vibration in a cesium watch.

**(d) Ampere :** One amp is that electric current which when passed through two parallel conductors of infinite length and of negligible cross section placed at unit distance apart produce a force of $2 \times 10^{-7}$ N/m.

**(e) Kelvin :**

**(i)** 1 Kelvin is equal to the $\left(\frac{1}{100}\right)^{th}$ part of the difference of melting point of ice and boiling point of water.

**(ii)** 1 Kelvin is equal to $\frac{1}{273.16}$th part of the thermodynamic temperature of triple point of water (273.16 K)

**(f) Candela :** It is the luminous intensity in a direction at right angles to surface of a black body at a temperature equal to melting point of platinum under pressure of 101, 325 $N/m^2$.

**(g) Mole :**

**(i)** Mole is that amount of substance contains as many elementary entities as there are oxygen atoms ($_8O^{16}$) in 0.016 kg of oxygen (old def.).

**(ii)** It is that amount of substance which contains as many elementary entities as there are carbon atoms $_6C^{12}$ in 0.012 kg of carbon.

**2.6 Supplementary Unit :**

**(a) Radian :** → 1 radian is the angle subtended by arc of length equal to the radius, at the centre of the circle.

**(b) Steradian :** It is defined as the solid angle subtended at the centre of the sphere by an arc of its surface equal to the square of radius of the sphere.

Solid angle $\Omega = \frac{A}{R^2}$

when $A = R^2$

$\Omega = 1$ steradian

**Example based on Units**

**Ex.3** The acceleration due to gravity is 9.80 $m/s^2$. What is its value in $ft/s^2$ ?

**Sol.** Because 1 m = 3.28 ft, therefore

$9.80\ m/s^2 = 9.80 \times 3.28\ ft/s^2$

$= 32.14\ ft/s^2$

**Ex.4** A cheep wrist watch loses time at the rate of 8.5 second a day. How much time will the watch be off at the end of a month ?

**Sol.** Time delay = 8.5 s/day

= 8.5 × 30 s/ (30 day)

$= 255$ s/month

$= 4.25$ min/month.

**Ex.5** If the units of force, energy and velocity in new system be 10N, 5J and 0.5 $ms^{-1}$ respectively, find the units of mass, length and time in that system.

**Sol.** Let $M_1$, $L_1$ and $T_1$ be the units of mass, length and time in SI and $M_2$, $L_2$ and $T_2$ the corresponding units in new system.

The dimensional formula for force is $(M^1L^1T^{-2})$

Hence the conversion formula for force becomes

$$n_2 = n_1 \left[\frac{M_1}{M_2}\right]^1\left[\frac{L_1}{L_2}\right]^1\left[\frac{T_1}{T_2}\right]^{-2}$$

Here $n_1 = 10$ N, $n_2 = 1$, Substituting we get

$$1 = 10 \left[\frac{M_1}{M_2}\right]\left[\frac{L_1}{L_2}\right]^2\left[\frac{T_1}{T_2}\right]^{-2} \quad ......(1)$$

The dimensional formula for work is $(M^1 L^2 T^{-2})$

$$n_2 = n_1 \left[\frac{M_1}{M_2}\right]^1\left[\frac{L_1}{L_2}\right]^2\left[\frac{T_1}{T_2}\right]^{-2}$$

$n_1 = 5$ J, $n_2 = 1$

Substituting values we get

$$1 = 5 \left[\frac{M_1}{M_2}\right]^1\left[\frac{L_1}{L_2}\right]^2\left[\frac{T_1}{T_2}\right]^{-2} \quad .......(2)$$

Similarly dimensional formula for velocity is $(M^0 L^1 T^{-1})$.

Hence, conversion formula for velocity is

$$n_2 = n_1 \left[\frac{M_1}{M_2}\right]^0\left[\frac{L_1}{L_2}\right]^1\left[\frac{T_1}{T_2}\right]^{-1}$$

Here $n_1 = 0.5\ ms^{-1}$, $n_2 = 1$,

Substituting values we get

$$1 = 0.5 \left[\frac{L_1}{L_2}\right]^1\left[\frac{T_1}{T_2}\right]^{-1} \quad ........(3)$$

Dividing (2) by (1), $1 = \frac{1}{2}\left[\frac{L_1}{L_2}\right]$,

$$L_2 = \frac{L_1}{2} = \frac{1}{2}\ m = 0.5\ m$$

Substituting value of $\left[\frac{L_1}{L_2}\right]$ in (3), we get

$$1 = 0.5 \times 2 \left[\frac{T_1}{T_2}\right]^{-1},$$

$$\frac{T_1}{T_2} = 1,\ T_2 = 1s$$

Substituting value of $\left[\frac{L_1}{L_2}\right]$ and $\left[\frac{T_1}{T_2}\right]$ in (1)

$$1 = 10 \left[\frac{M_1}{M_2}\right] \times 2 \times 1,$$

$$1 = 20 \left[\frac{M_1}{M_2}\right],$$

$M_2 = 20\ M_1$ as $M_1 = 1$kg, $M_2 = 20$ kg.

Hence units of mass, length and time are 20 kg, 0.5 m and 1 sec respectively

## 3. DIMENSIONS

**3.1.** Dimensions of a physical quantity are the powers to which the fundamental units of mass, length, time etc. must be raised in order to represent that physical quantity.

Dimensional formula = $[M^a L^b T^c Q^d]$ where a, b, c, d are the dimensions of M, L, T, Q respectively.

**Some Points About Dimensions :**

**(a)** The dimensions of a physical quantity do not depend upon system of units to represent that physical quantity.

**(b)** Pure numbers and pure ratio do not have any dimensions. i.e. these are dimension less, e.g. refractive index, relative density, relative permeability, cos $\theta$, $\pi$, strain etc.

**(c)** Similar dimension can be added or subtracted but it does not change the dimensions.

**(d)** For a physical equation to be correct dimensionally the dimension of all terms on two sides of the equation must be same. This is known as the principle of homogeneity of dimensions.

**(e)** Logarithmic functions as log x, $e^x$ is the dimension less quantity.

**(f)** Powers are dimension less.

**(g)** If we put the value of any physical quantity in any formula it seems unbalanced but reality is that it is balanced formula. Only appearance is unbalanced as :

$$S_n = u + \frac{a}{2}(2n - 1)$$

**(h)** The dimensions of two physical quantities may be same but the quantities need not be similar.

**(i)** Remember the following dimensional formula-

Force $= [M^1L^1T^{-2}]$

Energy $= [M^1L^2T^{-2}]$

**3.2. Uses of Dimension :** The uses of dimension are as given below.

**3.2.1** Homogeneity of dimensions in equation.
**3.2.2** Conversion of units
**3.2.3** Deducing relation among the physical quantities.

**3.2.1 Homogeneity of Dimensions in Equation :**

The dimensions of all the terms in an equation must be identical. This simple principle is called the principle of homogeneity of dimensions. This is the very useful method whether an equation may be correct or not. If the dimensions of all the terms are not same the equation must be wrong. Let us check the equation.

$$x = ut + \frac{1}{2}at^2$$

$$[x] = L$$

$$[ut] = \text{velocity} \times \text{time}$$

$$= \frac{\text{length}}{\text{time}} \times \text{time} = L$$

$$\left[\frac{1}{2}at^2\right] = [at^2]$$

$$= \text{acceleration} \times (\text{time})^2$$

$$= \frac{\text{velocity}}{\text{time}} \times (\text{time})^2$$

$$= \frac{\text{length / time}}{\text{time}} \times \text{time}^2 = L$$

Thus the equation is correct as far as the dimensions are concerned. The equation $x = ut + at^2$ is also dimensionally correct although this is an incorrect equation. So a dimensionally correct equation need not be physically correct but a dimensionally wrong equation must be wrong.

**3.2.2 Conversion of Units :**

When we choose to work with a different set of units for the base quantities, the units of all the derived quantities must be changed. Dimensions can be useful in finding the conversion factor for the unit of a derived physical quantity from one system to other.

**Ex.6** Convert the 1 pascal in to C.G.S. system

**Sol.** $P = \frac{F}{A}$

Thus, $[P] = \frac{[F]}{[A]} = \frac{MLT^{-2}}{L^2} = ML^{-1}T^{-2}$

So, 1 pascal $= (1kg)\,(1m)^{-1}\,(1s)^{-2}$

and

1 C.G.S. pressure $= (1g)\,(1\ cm)^{-1}\,(1s)^{-2}$

Thus

$$\frac{1\,\text{pascal}}{1\ \text{CGS pressure}}$$

$$= \left(\frac{1kg}{1gm}\right)\left(\frac{1m}{1cm}\right)^{-1}\left(\frac{1s}{1s}\right)^{-2}$$

$= (10^3)\,(10^2)^{-1} = 10$

or 1 pascal = 10 CGS pressure

One can work out the conversion factor for any derived quantity if the dimensional formula of the derived quantity is known.

**3.2.3 Deducing Relation among the physical quantities :** Some times dimensions can be used to deduce a relation between the physical quantities. If one knows the quantities on which a particular physical quantity depends and if one is given that this dependence is of product type. Method of dimension may be helpful to derive the relation.

**Ex.7** Derive the expression for the time period of a simple pendulum which depends on the length, mass, and gravitational acceleration.

**Sol.** $t \propto l^a m^b g^c \Rightarrow t = kl^a m^b g^c$

where k is a dimensionless constant and a, b and c are exponents which we want to evaluate. Taking the dimensions of both sides,

$T = L^a M^b (LT^{-2})^c = L^{a+c} M^b T^{-2c}$

Since the dimensions on both side must be identical we have

$a + c = 0$

$b = 0$

and $-2c = 1$

Giving $a = \frac{1}{2}$, $b = 0$

and $c = -\frac{1}{2}$

Putting this values in the equation $t = k\sqrt{\frac{\ell}{g}}$

**3.3. Limitations of the dimensional method :**

**(a)** First of all we have to know the quantities on which a particular physical quantity depends.

**(b)** Method works only if the dependence is of the product type (Not applicable for $x = ut + \frac{1}{2}at^2$)

**(c)** Numerical constants having no dimensions can not be deduce by the method of dimensions.

**(d)** Method works only if there are as many equations available as there are unknowns.

## Example based on Dimensions

**Ex.8** A gas bubble from an explosion under water, oscillates with a period T proportional to $p^a d^b E^c$ where p is the static pressure, d is the density of water and E is the total energy of explosion. Find the values of a, b and c.

**Sol.** As $T \propto p^a d^b E^c = k p^a d^b E^c$

k is dimensions on both sides

$M^0L^0T^1 = (M^1L^{-1}T^{-2})^a (M^1L^{-3})^b (M^1 L^2 T^{-2})^c$

$= M^{a+b+c} L^{-a-3b+2c} T^{-2a-2c}$

Applying principle of homogeneity of dimensions

$\therefore \quad a = \frac{-5}{6}, b = \frac{1}{2}, c = \frac{1}{3}$

**Ex.9** The velocity v of water waves depends upon their wavelength λ, density of water ρ and acceleration due to gravity, 'g'. Deduce by the method of dimensions the relation between these quantities.

**Sol.** Let the velocity v of water waves be given by

$v \propto \lambda^a \rho^b g^c$

$v = k\lambda^a \rho^b g^c$ ...... (1),

where k is dimensionless constant of proportionality and a, b, c are powers to be determined using principle of homogeneity of dimensions.

Taking dimensions on both sides of equation (1), we have

$M^0 L^1 T^{-1} = [M^0 L^1 T^0]^a [M^1 L^{-3} T^0]^b$

$[M^0 L^1 T^{-2}]^c = M^b L^{a-3b+c} T^{-2c}$

Comparing dimensions of mass, length and time, we get

$b = 0, c = \frac{1}{2}, a = \frac{1}{2}$

$\therefore \quad v = k\lambda^{1/2} \rho^0 g^{1/2}$, Thus $v \propto \sqrt{\lambda g}$

**Ex.10** If force F, length $\ell$, current I and time T are taken as fundamental base dimensions, then the dimension of permittivity $\varepsilon_0$ are –

(A) $F L^2 T^2 I^{-2}$ (B) $F^{-1} L^2 T^2 I^2$
(C) $F^{-1} L^{-2} T^2 I^2$ (D) $F^2 L^2 T^2 I^2$

**Sol.(C)** Since $F = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}$

we have $[\varepsilon_0] = \frac{[q_1 \times q_2]}{[F][r^2]} = \frac{(IT)(IT)}{FL^2}$

$= F^{-1} L^{-2} T^2 I^2$

The correct answer is (C)

**3.4. Application of dimensional analysis :**

**3.4.1 IN MECHANICS :**

(1) Write the definition or formula for the physical quantity.

(2) Replace M, L and T by the fundamental units of the required system to get the unit of physical quantity

**[a] Gravitational constant G :**

**I. Approach**

From Newton's law of gravitation we have

$$F = G\frac{m_1m_2}{r^2}$$

$$\Rightarrow \quad G = \frac{Fr^2}{m_1m_2}$$

$$[G] = \frac{[MLT^{-2}][L^2]}{[M][M]}$$

so its SI units is $\frac{m^3}{kgs^2}$ or $\frac{Nm^2}{kg^2}$

**II. Approach**

From the relation between G and g we have

$$g = \frac{Gm}{R^2}$$

$$\Rightarrow \quad G = \frac{gR^2}{m}$$

Substituting the dimensions of all physical quantities $[G] = \frac{[LT^{-2}][L^2]}{[M]}$

so its SI units is $\frac{m^3}{kgs^2}$ or $\frac{Nm^2}{kg^2}$

**[b] Plank constant h :**

**I. Approach**

According to constant h :

$$E = h\nu$$

$$\Rightarrow \quad h = \frac{E}{\nu}$$

Substituting the dimensions of known physical quantities :

$$[h] = \frac{[ML^2T^{-2}]}{T^{-1}} = M^1L^2T^{-1}$$

**II. Approach**

De Broglie

$$\lambda = \frac{h}{mv}$$

$$h = \lambda mv$$

Substituting the dimensions of known physical quantities :

$$[h] = [L]\,[M]\,[LT^{-1}]$$

$$[h] = [ML^2\,T^{-1}]$$

$$[h] = [ML^2\,T^{-1}]$$

**III. Approach**

Bohr's II Postulate

$$mvr = \frac{nh}{2\pi}$$

$$h = \frac{2\pi}{n} \times mvr$$

Substituting the dimensions of known physical quantities :

$$[h] = [mvr]$$

as $2\pi$ and n are dimensionless.

⇒ So SI unit of plank's constant is kg $m^2/s$. Which can also be written as (kg $m^2/s^2$) × s. But as kg $m^2/s^2$ is Joule so unit of h is Joule × sec. i.e. J-s

**[c] Coefficient of Viscosity η**

**I. Approach**

According to Newton's law

$$\Rightarrow \quad \eta = \frac{F}{A\left(\frac{dv}{dy}\right)}$$

Substituting the dimensional formulae of all other known physical quantities.

$$[\eta] = \frac{[MLT^{-2}]}{[L^2][LT^{-2}/L]}$$

**II. Approach**

Stoke's law $\quad F = 6\pi\,\eta rv$

$$\Rightarrow \quad \eta = \frac{F}{6\pi rv}$$

Substituting the dimensional formulae of all other known physical quantities.

$$[\eta] = \frac{[MLT^{-2}]}{[L][LT^{-1}]}$$

$$[\eta] = [ML^{-1}\,T^{-1}]$$

**III. Approach**

Poiseuille's formula

$$\frac{dV}{dt} = \frac{\pi pr^4}{8\eta\ell}$$

$$\Rightarrow \quad \eta = \frac{\pi pr^4}{8\ell\left(\frac{dV}{dt}\right)}$$

Substituting the dimensional formulae of all other known physical quantities.

$$[\eta] = \frac{[ML^{-1}T^{-2}][L^4]}{[L][L^3T]}$$

So SI or MKS unit of coefficient of viscosity is kg/m-s or (g/cm-s called poise in C.G.S. system)

**3.4.2 IN HEAT :**

**[a] Temperature :** In heat it is assumed to be a fundamental quantity with dimensions [θ] and unit Kelvin [K]. [How ever, ΔK = ΔC°].

**[b] Heat :** It is energy so it dimensions are $[ML^2T^{-2}]$ and SI Units Joule (J). Practical unit of heat is calorie (cal.) and

1 calorie = 4.18 joule

**[c] Coefficient of Linear Expansion** a **:**

It is defined as $\quad \alpha = \frac{\Delta L}{L\Delta\theta}$

i.e. $\quad [\alpha] = \frac{[L]}{[L][\theta]}$

i.e. $\quad [\alpha] = [\theta^{-1}]$

so its unit is $(C°)^{-1}$ or $K^{-1}$

**[d] Specific Heat C**

As $\quad \theta = mC\,\Delta\theta$

So $\quad C = \frac{\theta}{m\Delta\theta}$

i.e. $\quad [C] = \frac{[ML^{-2}T^{-2}]}{[M][\theta]}$

$$[C] = [L^2\,T^{-2}\,\theta^{-1}]$$

So its SI unit be J/kg-K while practical unit cal/gm-C°

**[e] Latent Heat L**

By definition

$$Q = ML$$

i.e. $\quad [L] = [ML^2\,T^{-2}]\,/\,[M]$

⇒ $\quad [L] = [L^2\,T^{-2}]$

So its SI unit be J/kg while practical unit cal/gm

**[f] Coefficient of Thermal conductivity K**

According to law of thermal conductivity

$$\frac{dQ}{dt} = KA\,\frac{d\theta}{dx}$$

or $[K] = \frac{[ML^2T^{-2}/T]}{[L^2][\theta/L]}$

$\Rightarrow$ $[K] = [MLT^{-3}\,\theta^{-1}]$

Its SI unit is W/mK while practical unit is cal/s – cm-C°.

**[g] Mechanical Equivalent of Heat J :**

According to I law of thermodynamics

$$W = JH$$

$$\Rightarrow \quad J = \frac{W}{H}$$

$$\Rightarrow \quad [J] = \frac{[ML^2T^{-2}]}{[ML^2T^{-2}]}$$

i.e. $[J] = [M^0\,L^0\,T^0]$

i.e. J has no dimensions. Its practical unit is J/cal and has value 4.18J/cal.

**[h] Boltzmann constant K** : According to kinetic theory of gases, energy of a gas molecule is given by

$$E = \frac{3}{2}KT$$

i.e. $[K] = \frac{[ML^2T^{-2}]}{[\theta]}$

i.e. $[K] = [ML^2T^{-2}\,\theta^{-1}]$

So its SI unit is $\frac{J}{K}$ and value $1.38 \times 10^{-23}\frac{J}{K}$.

**[i] Gas constant R:**

According to gas equation for perfect gas

$$PV = \mu RT$$

i.e. $[R] = \frac{[ML^{-1}T^{-2}][L^3]}{[\mu][\theta]}$

i.e. $[R] = [ML^2\,T^{-2}\,\theta^{-1}\,\mu^{-1}]$

So its SI unit is J/mol-K. While practical unit is cal/mol-K. It is a universal constant with value 8.31 J/mol-K or 2 cal/mol-K.

**[ j ] Vander Waal's constants a and b :**

Vander Waal's equation

$$\left(P + \frac{a}{V^2}\right)(V - b) = RT \qquad ....(1)$$

Vander waal's equation for $\mu$ mol is –

$$\left[P + \frac{\mu^2 a'}{V^2}\right][V - \mu b'] = \mu\,RT \qquad .... (2)$$

compare eq$^n$ (1) and (2)

$$\mu^2\,a' = a$$

and $\mu\,b' = b$

$$[a'] = \frac{[a]}{[\mu^2]}$$

and $[b'] = \frac{[b]}{[\mu]}$

$\Rightarrow$ $[a'] = [mL^5\,T^{-2}\,\mu^{-2}]$

and $[b'] = [L^3\,\mu^{-1}]$

Unit of a' and b' are $\frac{J - m^3}{mol^2}$ and $\frac{m^3}{mol}$ respectively.

### 3.4.3 IN ELECTRICITY :

**(a) Current I :** While dealing electricity we assume current to be a fundamental quantity and represent it by [A] with unit ampere (A)

**(b) Charge Q**

As $I = \frac{Q}{t}$

So $[Q] = [I]\,[t]$

$\Rightarrow$ $[Q] = [At]$

The SI unit of charge is A × s and is called coulomb (C).

**Note :**

(i) In MKSQ system charge is assumed to be fundamental quantity with dimension [Q] and unit coulomb. So in this system current will be derived with dimension $[QT^{-1}]$ and units coulomb / s which is ampere.

(ii) In CGS system there are two units of charge namely esu of charge frankline (Fr) and emu of charge. It is found that

1 coulomb = $3 \times 10^9$ esu of charge

$= \left(\frac{1}{10}\right)$ emu of charge.

**(c) Electric potential V :**

It is defined as $V = \frac{W}{q}$

So $[V] = \frac{[ML^2T^{-2}]}{[AT]}$

i.e. $[V] = [ML^2\,T^{-3}\,A^{-1}]$

So SI unit of potential is J/C and is called volt (V)

**(d) Electric intensity E :**

It is defined as

$$E = \frac{F}{q}$$

so $[E] = \frac{[MLT^{-2}]}{[AT]}$

$\Rightarrow \quad [E] = [MLT^{-3} A^{-1}]$

So SI unit of electric intensity is

$$\frac{\text{Newton}}{\text{Coulomb}} \rightarrow \frac{N-m}{c-m}$$

$$\xrightarrow{[N-m=J]} \frac{J}{c-m} \xrightarrow{\left[\frac{J}{C}=V\right]} \frac{V}{m}$$

**(e) Capacitance C**

It is defined as

$$q = CV$$

i.e. $\quad C = \frac{q}{V} = \frac{q^2}{W}$ [as $V = \frac{W}{q}$ ]

$$\Rightarrow \quad [C] = \frac{[AT]^2}{[ML^2T^{-2}]} = [M^{-1} L^{-2} T^4 A^2]$$

and its unit coulomb / volt is called farad.

**Note :** as W = qV joule / coulomb is volt → V

q = CV coulomb / volt is farad → F

**(f) Permittivity of free space $\varepsilon_0$ :**

According to coulomb's law

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}$$

$$\Rightarrow \quad [\varepsilon_0] = \frac{[q]^2}{[F][r]^2} = \frac{[A^2T^2]}{[MLT^{-2}][L^2]}$$

$$\Rightarrow \quad [\varepsilon_0] = [M^{-1} L^{-3} T^4 A^2]$$

and its unit is $\frac{\text{Coul}}{N-m^2} = \frac{\text{Coul}^2}{J-m}$ [as N – m = J]

$$\Rightarrow \frac{\text{Coul}}{\text{volt}-m}\left[\text{as}\frac{\text{Joule}}{\text{coul}} = \text{volt}\right]$$

$$\Rightarrow \frac{F}{m}\left[\text{as}\frac{\text{coul}}{\text{volt}} = \text{farad}\right]$$

**(g) Dielectric constant K or Relative permittivity $\varepsilon_r$:** As K → $\varepsilon_r = (\varepsilon/\varepsilon_0)$ so it has no units and dimensions.

**(h) Resistance R:**

According to ohm's law V = IR

$$\Rightarrow R = \frac{V}{I} = \frac{W}{qI} \text{ [as } V = \frac{W}{q}\text{ ]}$$

$$\Rightarrow [R] = \frac{[ML^2T^{-2}]}{[AT][A]} = [ML^2 T^{-3} A^{-2}]$$

and its unit volt / ampere is called ohm (Ω).

**(i) Resistivity or specific Resistance ρ :**

As $\quad R = \frac{\rho\ell}{\pi r^2}$

or $\quad \rho = \frac{\pi r^2 R}{\ell}$

$$\Rightarrow [\rho] = \frac{[R][L^2]}{[L]} \rightarrow [RL] \rightarrow [ML^3 T^{-3} A^{-2}]$$

and its unit is ohm-m or ohm-cm.

**(j) Coefficient of self induction L :**

According to definition of

$$L \text{ EMF} = \frac{Ldi}{dt}$$

or $\quad [L] = \frac{[W/q][T]}{[i]}$

i.e. $\quad [L] = \frac{[ML^2T^{-2}/AT][T]}{[A]}$

$$= [ML^2 T^{-2} A^{-2}]$$

and its unit $\frac{\text{volt}-s}{\text{amp}}$ → ohm × s is called Henry (H).

**(k) Magnetic flux φ:**

According to faraday's law of electro-magnetic induction

$$\text{E.M.F.} = \frac{d\phi}{dt}$$

So $[\phi] = [EMF]\,[T] = \left[\frac{W}{q}\right][T]$

$$\Rightarrow \quad [\phi] = [ML^2 T^{-2} / AT]\,[T]$$

$$= [ML^2 T^{-2} A^{-1}]$$

and its unit will be volt × s known as Weber (Wb.)

**(l) Magnetic Induction B :**

As force on a current element in a magnetic field is given by F = Biℓ sin θ

$$\Rightarrow \quad [B] = \frac{[F]}{[i][\ell]} = \frac{[MLT^{-2}]}{[A][L]} = [MT^{-2} A^{-1}]$$

and unit

$$\frac{N}{A\times m} \rightarrow \frac{J}{A\times m^2} \rightarrow \frac{C-V}{A-m^2} \rightarrow \frac{\text{volt}-s}{m^2} \rightarrow \frac{\text{weber}}{m^2}$$

and is called tesla (T).

**Note :** As φ = BA cos θ

$$\text{so } [B] = \frac{[\phi]}{[s]} = \frac{[ML^2T^{-2}A^{-1}]}{[L^2]}$$

$$= [MT^{-2} A^{-1}] \text{ and unit } \frac{\text{Weber}}{m^2}$$

**(m) Magnetic Intensity H :**

For Biot-savart law :

$$dB = \frac{\mu_0}{4\pi}\frac{Id\ell\sin\theta}{r^2}$$

$\Rightarrow$ $B = \mu_0 H$

$\Rightarrow$ $[H] = \left[\frac{Id\ell}{r^2}\right] = [AL^{-1}]$ and unit A/m.

| Physical Quantity | Dimensions | Units | | Relation |
|---|---|---|---|---|
| | | SI | CGS | |
| Mag. Flux $\phi$ | $[ML^2T^{-2}A^{-1}]$ | Weber | Maxwell | $1wb = 10^{18}$ Mx |
| Mag. Induction B | $[MT^{-2}A^{-1}]$ | Weber/$m^2$or tesla | Gauss | $1T = 10^4$ G |
| Mag. Intensity H | $[AL^{-1}]$ | Ampere/m | Orested | $1\frac{A}{m} = 4\pi \times 10^{-3}$ Oe<br>$1\frac{A}{m} = 4\pi \times 10^{-3}$ Oe<br>$= 4\pi \times 10^3$ G<br>$= 4\pi \times 10^{-7}\, t = \mu_0 T$ |

**(n) Magnetic Dipole moment M :**

As M = Nis

$\Rightarrow$ $[M] = [AL^2]$

and so unit $A - m^2$.

**(o) Permeability of free space $\mu_0$:**

Force per unit length between two parallel current carrying wires is given by :

$$\frac{dF}{dL} = \frac{\mu_0}{4\pi}\frac{I_1I_2}{r}$$

$\Rightarrow$ $[\mu_0] = \left[\frac{F}{I^2}\right]$

$\Rightarrow$ $[\mu_0] = \left[\frac{MLT^{-2}}{A^2}\right] = [ML\,T^{-2}\,A^{-2}]$

and its units

$$\frac{N}{A^2} \to \frac{J}{A^2m} \to \frac{Volt - s}{A - m} \to \frac{ohm \times s}{m} \to \frac{H}{m}$$

## Example based on Dimensions

**Ex.11** a, b are two different physical quantities with different dimensions which one of the following is correct ?

(A) a + b (B) a – b

(C) a/b (D) $e^{a/b}$

**Sol.** Hence correct answer is (C)

**Ex.12** $v = at + \frac{b}{ct + v}$ Find out the value of a, b, c-

(A) $L^2 T^{-1}, L^2 T^{-1}, L^1 T^{-2}$

(B) $L^1 T^{-2}, L^1 T^{-1}, L^1 T^{-2}$

(C) $L^1 T^{-2}, L^2 T^{-1}, L^1 T^{-2}$

(D) $L^1 T^{-2}, L^2 T^{-1}, L^1 T^{-3}$

**Sol.** $[LT^{-1}] = a\,[T] + \frac{b}{C[T] + M^0L^1T^{-1}}$

$a = L^1 T^{-2}$

$b = L^2 T^{-1}$

$c = L^1 T^{-2}$

Hence correct answer is (C).

**Note :** It is difficult to remember dimensions of all the quantities so if we want to find out dimensions of any quantity we should use the formula in which whatever the quantities we are using, that should be go towards fundamental quantities.

**Ex.13** Find out the dimension of L (Inductance)-

(A) $M^1 L^2 T^{-2} A^{-2}$ (B) $M^1 L^3 T^{-2} A^{-2}$

(C) $M^1 L^2 T^{-3} A^{-2}$ (D) $M^1 L^2 T^{-2} A^{-1}$

**Sol.** We know that $L = \frac{\mu N^2 A}{\ell}$

But if we do not remember the dimension of $\mu$ then it will be difficult for us to determine the dimension of L.

Therefore we will use $U = \frac{1}{2}LI^2 \Rightarrow L = \frac{2U}{I^2}$

(where U : energy of inductor)

$\because U = M^1 L^2 T^{-2}$

$[M^a L^b T^c A^d] = \frac{[M^1L^2T^{-2}]}{[A^2]}$

$\therefore [M^a L^b T^c A^d] = [M^1 L^2 T^{-2} A^{-2}]$

$\therefore a = 1, b = 2, c = -2, d = -2.$

Hence correct answer is (A)

**Note** :

Similarly find out the dimension of R & C

From $P = I^2R = \text{Power} = \frac{\text{Work}}{\text{Time}}$

and $U = \frac{1}{2}CV^2 = \text{Energy}$.

**Ex.14** To convert the physical quantity of one unit to physical quantity of another unit.

**Sol.** 1 force of M.K.S. = x force C.G.S.

$\Rightarrow 1\,[M^1 L^1 T^{-2}]$ M.K.S.= x $[M^1 L^1 T^{-2}]$ C.G.S

$\Rightarrow (kg)^1 (m)^1 (sec)^2 = x\,(gm)^1 (cm)^1 (sec)^{-2}$

$\Rightarrow (1000)^1 (gm)\,(100)^1 (cm)\,(sec)^{-2}$

$= x\,(gm)^1 (cm)^1 (sec)^{-2}$

$x = 10^5$

So 1 force of M.K.S.= $10^5$ force C.G.S.

## 4. ORDER OF MAGNITUDES

**(a)** Normally decimal is used after first digit using powers of ten,

**Example :** 3750 m will be written as $3.750 \times 10^3$m

**(b)** The order of a physical quantity is expressed in power of 10 and is taken to be 1 if $\le (10)^{1/2} = 3.16$ and 10 if > 3.16

**Example :** speed of light = $3 \times 10^8$, order = $10^8$

Mass of electron = $9.1 \times 10^{-31}$, order = $10^{-30}$

**(c)** **Significant digits :** In a multiplication or division of two or more quantities, the number of significant digits in the answer is equal to the number of significant digits in the quantity which has the minimum number of significant digit.

**Example :** 12.0/7.0 will have two significant digits only.

**(d)** The insignificant digits are dropped from the result if they appear after the decimal point. They are replaced by zeroes if they appear to the left of two decimal point. The least significant digit is rounded according to the rules given below.

**Rounding off :** If the digit next to one rounded as more then 5, the digit to be rounded is increased by 1 ; if two digit next to the one rounded is less than 5, the digit to be rounded is left unchanged, if the digit next to one rounded is 5, then the digit to be rounded is increased by 1 if it is odd and is left unchanged if it is even.

**(e)** For addition and subtraction write the numbers one below the other with all the decimal points in one line now locate the first column from left that has doubtful digits. All digits right to this column are dropped from all the numbers and rounding is done to this column. The addition and subtraction is now performed to get the answer.

**(f)** Number of 'Significant figure' in the magnitude of a physical quantity can neither be increased nor decreased.

**Example :** If we have 3.10 kg than it can not be written as 3.1 kg or 3.100 kg.

## Example based on Significant Digits

**Ex.15** Round off the following numbers to three significant digits

(A) 15462 (B) 14.745

(C) 14.750 (D) $14.650 \times 10^{12}$.

**Sol.**(A) The third significant digit is 4. This digit is to be rounded. The digit next to it is 6 which is greater than 5. The third digit should, therefore be increased by 1. The digits to be dropped should be replaced by zeros because they appear to the left of the decimal. Thus, 15462 becomes 15500 on rounding to three significant digits.

(B) The third significant digit in 14.745 is 7. The number next to it is less than 5. So 14.745 becomes 14.7 on rounding to three significant digits.

(C) 14.750 will become 14.8 because the digit to be rounded is odd and the digit next to it is 5.

(D) $14.650 \times 10^{12}$ will become $14.6 \times 10^{12}$ because the digit to be rounded is even and the digit next to it is 5.

**Ex.16** Evaluate $\frac{25.2 \times 1374}{33.3}$. All the digits in this expression are significant.

**Sol.** We have $\frac{25.2 \times 1374}{33.3} = 1039.7838\ .....$

Out of the three numbers given in the expression 25.2 and 33.3 have 3 significant digits and 1374 has four. The answer should have three significant digits. Rounded 1039.7838 .... to three significant digits, it becomes 1040.

Thus , we write.

$$\frac{25.2 \times 1374}{33.3} = 1040.$$

**Ex.17** Evaluate 24.36 + 0.0623 + 256.2

**Sol.**

24.36
0.0623
256.2

Now the first column where a doubtful digit occurs is the one just next to the decimal point (256.2). All digits right to this column must be dropped after proper rounding. The table is rewritten and added below

24.4
0.1
256.2
———
280.7 The sum is 280.7

## 5. FRACTIONAL AND PERCENTAGE ERRORS

**5.1** If Δx is the error in measurement x, then

Fractional error $= \frac{\Delta x}{x}$

Percentage error $= \frac{\Delta x}{x} \times 100$

Percentage error in experimental measurement

$$= \frac{\text{experimental value} \sim \text{Standard value}}{\text{Standard value}}$$

**5.2 PROPAGATION OF ERROR (ADDITION AND SUBTRACTION) :**

Let error in x is ± Δx, and error in y is ± Δy, then the error in x + y or x – y is ± (Δx + Δy). The errors add.

**5.3 MULTIPLICATION AND DIVISION :**

Let errors in x, y, z are respectively ± Δx, ± Δy and ± Δz. Then error in a quantity f (defined as)

$f = \frac{x^a y^b}{z^c}$ is obtained from the relation

$$\frac{\Delta f}{f} = |a| \frac{\Delta x}{x} + |b| \frac{\Delta y}{y} + |c| \frac{\Delta z}{z}.$$

The fraction errors (with proper multiples of exponents) add. The error in f is ± Δf.

**Important Points :**

**(a)** When two quantities are added or subtracted the absolute error in the find is the sum of the absolute error in the quantities.

**(b)** When two quantities any multiplied or divided, the fractional error in the result is the sum of the fractional error in the quantities to be multiplied or to be divided.

**(c)** If the same quantity x is multiplied together n times (i.e. $x^n$), then the fractional error in $x^n$ is n times the fractional error in x,

i.e. $\pm n \frac{\Delta x}{x}$

**Example based on Error**

**Ex.18** In an experiment to determine acceleration due to gravity by simple pendulum, a student commit positive error in the measurement of length and 3% negative error in the measurement of time period. The percentage error in the value of g will be-

(A) 7% (B) 10%
(C) 4% (D) 3%

**Sol.** We know $T = k\sqrt{\frac{\ell}{g}}$

$\therefore T^2 = k'\left(\frac{\ell}{g}\right) \Rightarrow g = k' \frac{\ell}{T^2}$

$$\frac{\Delta g}{g} \times 100 = \frac{\Delta \ell}{\ell} \times 100 + \frac{2\Delta T}{T} \times 100$$

$$= 1\% + 2 \times 3\% = 7\%$$

Hence correct answer is **(A).**

| | |
|---|---|
| Capacitance | Ohm × second |
| Inductance | $coul^2\ joul^{-1}$ |
| Magnetic indication | coulomb $(volt)^{-1}$ |
| | Volt – sec $(Ampere)^{-1}$ |
| | Newton $(ampere–m)^{-1}$ |

# SOLVED EXAMPLES

**Ex.1** If velocity, force and time are taken to be fundamental quantities, find dimensions formula for (a) mass.

(A) $K^{-1} V^{-1} FT$ (B) $K V^{-1} FT$

(C) $K V F^{-1} T^{-1}$ (D) $K V^{-1} F^{-1} T$

**Sol.** Let the mass is represented by M then

$$M = f(V, F, T)$$

Assuming that a function is product of power functions of V, F and T

$$M = KV^x F^y T^z$$

Where K is a dimension less constant of proportionality. The above equation dimensionally becomes.

$$[M] = [LT^{-1}]^x [MLT^{-2}]^y [T]^2$$

i.e. $[M] = [M^y] [L^{x+y} T^{-x-2y+z}]$

So equation becomes

$$[M] = [M^y L^{x+y} T^{-x-2y+z}]$$

For dimensionally correct expression,

$y = 1$, $x + y = 0$ and $-x - 2y + z = 0$

$\Rightarrow x = -1$, $y = 1$ and $z = 1$.

therefore $M = KV^{-1} FT$.

Hence correct answer is (B).

**Ex.2** 5 litre of benzene will weigh more in summer or winter ?

(A) Summer (B) Winter

(C) Equal in both (D) None of these

**Sol.** We know that weight = w = mg = $\rho Vg$

Here value V ( = 5 lit) and acceleration due to gravity g ( = 9.8 $m/sec^2$) are constant

$$w \propto m \propto \rho$$

Now with size in temp due to thermal expansion density decrease, so that weight of 5 litre benzene will be less in summer that in winter.

Hence correct answer is (B).

**Ex.3** Column I gives three physical quantities. Select the appropriate units for these from choices given in column. Some of the physical quantities may have more than one choice.

**I** **II**

**Sol.** (I) $q = CV$ i.e. $[C] = \left[\frac{q}{V}\right]$

so $[C] = [M^{-1} L^{-2} T^4 A^2]$

$U = \frac{1}{2} LI^2$ i.e. $[L] = \left[\frac{U}{I^2}\right]$

so $[L] = [M^1 L^2 T^{-2} A^{-2}]$

$F = Bil$ since i.e. $[B] = \left[\frac{F}{i\ell}\right]$

so $[B] = [MT^{-2} A^{-1}]$

(II) Now the dimensions from given units are

ohm × sec $\equiv [R][T] \longrightarrow$

$[ML^2 T^{-3} A^{-2}] [T] = [ML^2 T^{-2} A^{-2}]$

$coul^2 - volt^{-1} \equiv \left[\frac{q^2}{W}\right] \rightarrow \frac{[A^2T^2]}{[ML^2T^{-2}]}$

$= [M^{-1} L^{-2} T^4 A^2]$

coul $(volt)^{-1} \equiv \rightarrow \frac{[A^2T^2]}{[ML^2T^{-2}]}$

$= [M^{-1} L^{-2} T^4 A^2]$

Newton $(amp\text{-}m)^{-1} \equiv \left[\frac{F}{i\ell}\right] \rightarrow \frac{[MLT^{-2}]}{[AL]}$

$= [M T^{-2} A^{-1}]$

volt sec $(amp)^{-1} \equiv \left[\frac{W \times T}{q \times A}\right] \rightarrow \frac{[ML^2T^{-2}][T]}{[ATA]}$

$= [ML^2 T^{-2} A^{-2}]$

Comparing dimensions of II with F we find that capacitance has unit [$coulomb^2$–$joule^{-1}$ and coulomb $(volt)^{-1}$] inductance has units [ohm-sec and volt-sec $(ampere)^{-1}$] and magnetic induction has unit [Newton $(ampere\text{-}m)^{-1}$]

**Ex.4** A certain physical quantity is calculated from the formula $\frac{\pi}{3}(a^2 - b^2)h$ where h, a and b are all length. The quantity being calculated is-

(A) Velocity (B) Length

(C) Area (D) Volume

**Sol.** Given quantity is = $\frac{\pi}{3}(a^2 - b^2)h$

dimension of h = [L]

dimension of $a^2 - b^2 = [L^2 - L^2] = L^2$

Therefore the dimension of the given quantity are $[L^3]$. Thus the quantity being measured is volume. **Ans. (D)**

**Ex.5** When a current of 2.3 ± 0.5 ampere flows through a wire, it develops a potential difference of 20 ± 1 volt. Find the resistance of the wire.

(A) 6 ± 3 (B) 7 ± 2

(C) 8 ± 2 (D) 9 ± 3

**Sol.** $R = \frac{V}{I} = \frac{20 \pm 1}{2.5 \pm 0.5} = 8 \pm \Delta R$

the error in the measurement is

$= \frac{\Delta V}{V} + \frac{\Delta I}{I} = \frac{1}{20} + \frac{0.5}{2.5} = 0.05 + 0.2$

$= 0.25$

$\Delta R = 0.25\, R = 0.25 \times 8 = 2$

Thus the resistance of the wire with the error is = 8 ± 2 ohm.

Hence correct answer is (C).

**Ex.6** In an experiment the values of two resistances were measured to be as given below. $R_1 = 5.0 \pm 0.2$ ohm and $R_2 = 10.0 \pm 0.1$ ohms. Find their combined resistance in (i) Series (ii) Parallel.

**Sol.** (i) In series

$R \pm \Delta R = (R_1 + R_2) \pm (\Delta R_1 + \Delta R_2)$

$R = [(5 + 10) \pm 0.3] = [15 \pm 0.3]\ \Omega$

or R = [15 ± 2%] because the error in percentage is = $\frac{0.3 \times 100}{15} = 2\%$

(ii) In parallel

$R = \frac{R_1 R_2}{R_1 + R_2} = \frac{10}{3} = 3.3\ \Omega$

For error calculations, we proceed as follows Taking log we get

$\log R = \log R_1 + \log R_2 - \log (R_1 + R_2)$

Differentiating we get,

$$\frac{\Delta R}{R} = \frac{\Delta R_1}{R_1} + \frac{\Delta R_2}{R_2} - \frac{\Delta(R_1 + R_2)}{R_1 + R_2}$$

$$\therefore \frac{\Delta R}{R} \times 100 = \frac{\Delta R_1}{R_1} \times 100 + \frac{\Delta R_2}{R_2} \times 100 - \frac{\Delta R_1 \times 100}{(R_1 + R_2)} - \frac{\Delta R_2 \times 100}{(R_1 + R_2)}$$

Maximum error in R (make all terms positive)

$$\therefore \frac{\Delta R}{R} \times 100 = \frac{\Delta R_1}{R_1} \times 100 + \frac{\Delta R_2}{R_2} \times 100 + \frac{\Delta R_1 \times 100}{(R_1 + R_2)} + \frac{\Delta R_2 \times 100}{(R_1 + R_2)}$$

or $\frac{\Delta R}{R} \times 100 = \frac{0.2}{5} \times 100 = \frac{0.1 \times 100}{10}$ $\frac{0.2 \times 100}{15} + \frac{0.1 \times 100}{15} = 4\% + 1\% + 2\% + 7\%$

$\therefore R = 3.3 \pm 7\%\ \Omega.$